# PD-361-S.E.-CV-19

## M.A./M.Sc. MATHEMATICS ($3^{red}$ Semester)

Examination, Dec.-2020

Paper-II

PARTIAL DIFFERENTIAL EQUATIONS, MACHANICS & GRAVITAION-I

Time : Three Hours] [Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

**Section-A**

1. **Answer the following very short answer type questions:-** 1x10=10
   (a) Write formula for Green's function in the form of ∅.
   (b) Solution of wave equation for n=3 is called.
   (c) Write complete integral for Clairaut's equation $x - Du + f(Du) = u(x,a)$
   (d) Write Hamilton Jacobi equation.
   (e) Write the relation between Hamilton H and Lagrangian function L.
   (f) Write porous medium equation.
   (g) Write formula of Gauss theorem for surface integral.
   (h) Write Lax oleinik formula.
   (i) Write Laplace transform for $(Sint - Cost)^2$
   (j) Write the value of $L^{-1}\left(\frac{1}{\sqrt{P}}\right)$
2. **Answer the following short answer type questions:-** 2x5=10
   (a) Find complete integral of Hamilton Jacobi equation.
   (b) Define envelope of the function $u(x,a)$
   (c) Write work done by self attracting system.
   (d) Find the value of $L\left(\frac{Cos\sqrt{t}}{\sqrt{t}}\right)$
   (e) Find the value of $L^{-1}\left(\frac{P}{(P+1)^5}\right)$

**Section-B** 12x5=60

Answer long type questions.

3. Derive the fundamental solution of Laplace equation.

   OR

   State and prove mean Value property for heat equation.
4. State and prove characteristic ordinary differential equation.

   OR

   State and prove Hamilton ordinary differential equation.
5. For Asymptotic in $L^{\infty}$ norm prove that there exist a constant C such that
   $$|u(l,t)| \le \frac{C}{\sqrt{t}}$$

   OR

   (a) State and prove Hopf Lax formula.
   (b) Use separation of variables solve the equation $ut - \Delta(u^r) = o \text{ in } R^N X(0,\infty)$
6. To find the attraction of thin uniform circular disc at external point whose radius become infinite.

   OR

   State and prove Poisson equations using normal attraction.
7. Using Laplace Transform to solve $(D+2)^2 Y = 4e^{-2t}$, when $y(0) = -1$ and $y'(0) = 4$

   OR

   Using Fourier transform prove that for $u, v \in L^2(R^n)$
   (a) $\int_{R^n} \hat{u}\hat{v}dx = \int_{R^n} \hat{u}\hat{\bar{v}}.dy$ (b) $\hat{D^{\alpha}u} = (iy)^{\alpha}.\hat{u}$, where $\hat{u}, \hat{v}$ are Fourier Transform