PD-365

(513) M.A./M.Sc. MATHEMATICS (THIRD SEMESTER)

Examination Dec. 2020

Compulsory/Optional

Group-

Paper-VI

Title of Paper- **FLUID MECHANICS-I** [Maximum Marks- 80

Time: Three Hours [Minimum Passing Marks- 29

नोटः दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note: Answer from Both the Section as Directed. The Figures in the right hand margin indicate marks.

Section- A

1. **Answer the following question:** **1x10**
   (a) Write Lagrangian equation of continuity.
   (b) Define path lines.
   (c) Write the condition for irrational motion of fluid.
   (d) What is complex potential?
   (e) Define the strength of two dimension source.
   (f) Define the image of two dimension source and sinks.
   (g) If W is the complex potential then the magnitude of velocity is ....
   (h) If stream function ψ and velocity potential function ϕ are functions of γ, θ then $\frac{\partial \phi}{\partial \gamma} =$ ....
   (i) If k be the constant circulation about the cylinder then the suitable form of ϕ is ....
   (j) Define equipotential surface and write the equation of equipotential surface.
2. **Answer the following question:** **2x5**
   (a) Determine the acceleration at the point (2,1,3) at $t = 0.5Sec$ if $u = yz+t$, $v = xz-t$ and $w = xy$.
   (b) Define vortex line and obtain its differential equation.
   (c) If the velocity of an incompressible fluid at the point (x,y,z) is given by $(\frac{3xz}{\gamma^5}, \frac{3yz}{\gamma^5}, \frac{3z^2-\gamma^2}{\gamma^5})$, prove that the liquid motion is possible.
   (d) What arrangement of sources and sinks will give rise to the function $w = \log (z- \frac{a^2}{z})$?
   (e) Define strength of three dimensional source and if q_γ the radial velocity at a distance γ from the source, then find the value of q_γ.

Section- B

Answer the following questions: **12x5**

3. Derive the equation of continuity in polar co-ordinates.

OR

If every particle moves on the surface of a sphere, prove that the equation of continuity is

$$\frac{\partial \delta}{\partial t} cos\theta + \frac{\partial}{\partial \theta}(\delta w\ cos\theta) + \frac{\partial}{\partial \emptyset}(\delta w' cos\theta) = 0$$

δBeing the density,$\theta,\emptyset$ the latitude and longitude of any element and w and w' the angular velocities of element in latitude and longitude respectively.

4. An infinite mass of fluid is acted on by a force $\mu\gamma^{-3/2}$ unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $\gamma = c$ in it, show that cavity will be filled up after an internal of time $(\frac{2}{5\mu}) c\ ^5/_4$

OR

A spherical hollow of radius a initially exists in a infinite fluid subject to constant pressure at infinity. Show that the pressure at distance γ from the center when the radius of the cavity is x is to the pressure at infinity as

$$3x^2\gamma^4 + (a^2 - 4x^3)\gamma^3 - (a^3 - x^3)x^3 : 3x^2\gamma^4$$

5. Stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D and d; if V and μ be the corresponding velocities of the stream, and if the motion be supposed to be that of divergence from the vertex of the cone, prove that

$$\frac{\mu}{v} = \frac{D^2}{d^2} e^{(\mu^2 v^2)/2k}$$

OR

Two equal closed cylinders of height C with their bases in the same horizontal plane are filled one with water and the other with air of such a density as to support a column h of water, h being less than C. If a communication be opened between them at their bases then show that the height x, to which the water rises is given by the equation $cx - x^2 + ch \log\left(\frac{c-x}{c}\right) = 0$.

6. A source of strength m, placed outside the circle, find the image of source.

OR

In the case of the motion of liquid in a part of a plane bounded by a straight line due to a source in the plane, prove that if $m\delta$ is the mass of fluid (of density δ) generated at the source per unit of time the pressure on the length 2ℓ of the boundary immediately opposite to the source is less than that on an equal length at a great distance by

$$\frac{1}{2}\frac{m^2\delta}{\pi^2}\left\{\frac{1}{c}\tan^{-1}\frac{1}{c} - \frac{\ell}{\ell^2 + c^2}\right\}$$

Where C is the distance of the source from the boundary.

7. State and prove theorem of Blasius.

OR

The space between two infinitely long coaxial cylinders of radii a and b respectively is filled with homogeneous liquid of density δ and the inner cylinder is suddenly moved with velocity U perpendicular to the axis, the outer one being kept fixed, show that the resultant impulsive pressure on a length ℓ of the inner cylinder is

$$\pi\delta a^2 \ell \frac{b^2 + a^2}{b^2 - a^2} U$$