PC – 485 CV-19

M.A./M.Sc. (Fourth Semester)

Examination June2020

MATHEMATICS

Compulsory/Optional

Paper-VI

# FLUID MECHANICS

Time : Three Hours] [Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from **both** the Sections as directed. The figures in the right-hand margin indicate marks.

## Section-A

1. Answer the following questions:- [1x10=10]

(a) The stream function due to cylinder which is moving with velocity U and V parallel to r and y –aris and rotating with angular velocity w is –

(i) $\psi = V_{xy} - U + \frac{1}{2} w(x^2 + y^2) + \text{const.}$

(ii) $\psi = V_x - U_y + \frac{1}{2} w(x^2 + y^2) + \text{const.}$

(iii) $\psi = UV - \frac{1}{2} wxy + \text{const.}$

(iv) $\psi = UV - \frac{1}{w}(x^2 + y^2) + \text{const.}$

(b) The kinetic energy T of the liquid on the boundary of the cylinder-

(i) $-\frac{1}{2}\rho \int \phi d\psi$

(ii) $-\frac{1}{2}\rho \int d\psi$

(iii) $-\frac{1}{2}\rho \int d\phi$

(iv) $\frac{1}{2}\rho \int d(\phi\psi)$

(c) The velocity potential of a rotating elliptic cylinder is $\phi$ =

(i) wxy

(ii) $-w(a^2 - b^2)\, xy$

(iii) $-w\left(\frac{a^2-b^2}{a^2+b^2}\right) xy$

(iv) $-w\,\frac{a^2+b^2}{(a^2-b^2)\,xy}$

(d) $\phi$ due to three dimensional source mat the origin is-

(i) $va^3 r \cos\theta$

(ii) $1/2 va^3 \sin\theta$

(iii) $-1/2 va^3/r \sin\theta$

(iv) $½ va^3/r^2 \cos\theta$

(e) The impulse I necessary to produce the velocity U in the inner sphere of mass $M_2$ then I – MU =

(i) $\iint \overline{w} \cos\theta ds$

(ii) $\iint w \cos^2\theta ds$

(iii) $\iint \overline{w} \sin\theta ds$

(iv) $\iint |w| \sin^2\theta ds$

(f) The equation of continuity in cylindrical coordinates is-

(i) $\frac{\partial\phi}{\partial t} + \frac{1}{r}\frac{\partial x}{\partial t} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial}{\partial z}(\rho w) = 0$

(ii) $\frac{\partial\phi}{\partial t} + \frac{1}{r}\frac{\partial y}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u) + \frac{1}{r}\frac{\partial}{\partial}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

(iii) $\frac{1}{r}\frac{\partial\phi}{\partial t} + \frac{\partial}{\partial r}(\rho r u) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

(iv) $r\frac{\partial\phi}{\partial t} - \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta}(\rho w) + \frac{\partial}{\partial z}(\rho w) = 0$

(g) In usual notation, the stoke's stream function for a simple source on the axis of r is

(i) mx/r (ii) m sinθ (iii) $mx/r^2$ (iv) mx

(h) In the vortex motion the velocity field at a point $P(\vec{r})$ is $\vec{q}(\vec{r},t)$ =

(i) $\frac{1}{4\pi} \iiint \frac{\vec{\Omega}(\vec{r}',t)}{|\vec{r}-\vec{r}'|} dv'$

(ii) $\text{curl}\, \frac{1}{4\pi} \iiint \frac{\vec{\Omega}(\vec{r}',t)}{|\vec{r}-\vec{r}'|} dv'$

(iii) div $\frac{1}{4\pi} \iiint \frac{\Omega'(\vec{r'},t)}{|\vec{r}-\vec{r'}|}$ dv'

(iv) curl $\iiint \Omega'(\vec{r'}, t)$ dv'

(i) Which of the following represent wave equations-

(i) $\partial^2 y/\partial x^2 = 1/c^2\, \partial y/\partial t$

(ii) $\partial^2 y/\partial x^2 = 1/c^2\, \partial^2 y/\partial x\partial t$

(iii) $\partial^2 y/\partial x^2 = 1/c^2\, \partial^2 y/\partial t^2$

(iv) $\partial y/\partial x = 1/c\, \partial^2 y/\partial t^2$

(j) The potential for a progressive wave in a deep canal of uniform depth is w =

(i) $ace^{-i(mz-nt)}$ (ii) $e^{i(mz-nt)}$ (iii) $ace^{(mz-nt)}$ (iv) $e^{(mz-nt)}$

2. Answer the following questions- [2x5=10]

(a) Find kinetic energy T when the elliptic cylinder moves with velocity U parallel to x – axis.

(b) Show that a sphere projected in a liquid under gravity describes a parabola of latus rectum $\left(\frac{2\sigma+\rho}{\sigma-\rho}\right)\frac{w^2}{g}$,

where σ and ρ are the densities of the sphere and the liquid and W is the horizontal velocity.

(c) Find the necessary and sufficient condition that the vortex lines may be at right angles to the stream lines.

(d) Discuss the progressive waves reduced to a steady motion.

(e) What are capillary waves?

**Section-B**

Answer the following long – answer type questions- [12x5=60]

3. In the two dimensional irrotational motion of a liquid streaming past a fixed elliptic disc $r^2/a^2 + y^2/b^2 = 1$, the velocity at infinity being parallel to the major axis and equal to V. If $x + iy = c\cosh(\xi+i\eta)$, $a^2 - b^2 = c^2$ and $a=c\cosh\alpha$, $b = c\sinh\alpha$, then prove that the velocity at any point is

$$q2 = v2(a + b/a - b).\frac{(\sinh 2(\xi - \alpha)+\sin 2\eta)}{\sinh 2\xi+\sin 2\eta}$$

Further prae that it has its maximum value V(a+b)/a at the end of the minor axis.

**OR**

A thin shell in the form of an infinitely long effiptic cylinder, semi-areas a and b, is rotating about its axis in an infinite liquid otherwise at rest. It is filled with the same liquid. Prove that the ratio of the kinetic energy of the liquid inside to that of the liquid outside is $2ab(a^2+b^2)$.

4. liquid of density ρ fille the space between a solid sphere of radius a and density ρ' and a fired concentric spheried envelop of radius b; prove that the work done by an impulse which starte the solid sphere with velocity V is $\frac{1}{3}\pi a v2 \left\{2\rho' + \frac{2a^3+b^3}{b^3-a^3}\right\}$

**OR**

Obtain values of Stoke's Stream Function in following cases:

(i) A simple source on the axis of x.

(ii) A doublet along the axis of x.

(iii) A uniform line source along the axis of x.

5. Prove that the product of the cross section and vorticity at any point on a vortex filament is constant along the filament and for all time when the body forces are conservative and the pressure is a single valued function of density only.

**OR**

Show that the motion due to a set of line vortices of strength k at z = na(n=0.1.2.......) is given by the equation W = Ik/2π log sin(πz/a) also find velocity components and steam lines.

6. Explain Karman street.

**OR**

Show that the total energy of progressive waves is half kinetic energy and half potential energy.

7. Find the dispersion equation for plane sound waves in air, accounting for viscosity and heat conduction.

**OR**

Prove that in a uniform heavy liquid of depth k, thre is not more than one wave length corresponding to any given velocity, and that any velocity less than $\sqrt{2g}$ is the velocity of the same wave.